

Enhancing mathematical models for phosphate transport by machine learning

Mohammed Seaid

Department of Engineering, University of Durham, South Road,
DH1 3LE, United Kingdom

International Water Research Institute, University Mohammed VI
Polytechnic, Benguerir, Morocco

Abstract

Mine transport in its hydraulic form has been attracting more and more industries. In this work, we discuss challenges related to the transport of Phosphate in its slurry form. This dynamic is highly non-Newtonian and very little knowledge about its rheological properties is available. For that purpose, we examine the ability of machine learning tools to predict this information using available real data. This dataset includes information about rheology but also many physico-chemical parameters such as the density, the solid concentration, the particle size distribution among others. Three well-known machine learning algorithms are tested. Despite an important overfitting observed due to the limited amount of data, the results are encouraging and pave the way towards predicting with reasonable accuracy the hydraulic of the Phosphate slurry fluid in pipelines. We also introduce a new mathematical model for modelling and simulation of phosphate slurry using non-Newtonian incompressible fluids. These equations are coupled to an advection-diffusion equation for the slurry transport and the obtained concentration is used for the fluid viscosity. Using a fixed-point method we prove the existence and uniqueness of the weak solution for the considered problem. The proposed scheme uncouples the computation of velocity and concentration using the fixed-point iteration and we theoretically establish its convergence towards the unique solution of the considered model. Numerical results obtained for several test examples are presented to verify the theoretical analysis and to assess the performance of the proposed model.

References

- [1] M. El-Amrani, A. Obbadi, M. Seaid, D. Yakoubi, *Error estimates for a viscosity-splitting scheme in time applied to non-Newtonian fluid flows*, Computer Methods in Applied Mechanics and Engineering, Vol. 419, pp. 116639 (2024)
- [2] N. El Mocayd, M. Seaid, *Rheology data-driven machine learning models for phosphate slurry pipeline in Morocco*, AIP Conference Proceedings, Vol. 3034 (1), pp. 110004 (2023)
- [3] N. El Mocayd, M. Seaid, *Data-driven polynomial chaos expansions for characterization of complex fluid rheology: Case study of phosphate slurry*, Reliability Engineering & System Safety, Vol. 216, pp. 107923 (2021)

Novel theoretical and numerical advances in the
field of fractional calculus with applications to
life and social sciences

Khalid Hattaf

Equipe de Recherche en Modélisation et Enseignement des Mathématiques (ERMEM),
Centre Régional des Métiers de l'Éducation et de la Formation (CRMEF),
Casablanca, Morocco

Abstract

In recent years, fractional calculus has witnessed significant advances in both its theoretical foundations and practical applications across various fields of science and engineering. This work presents novel theoretical and numerical results by establishing new stability criteria and developing numerical methods for solving fractional differential equations (FDEs) involving the generalized Hattaf fractional (GHF) derivative, which encompasses many types of fractional operators with non-singular kernels. Moreover, the obtained results are applied to some dynamical systems arising from the life and social sciences.

References

- [1] K. Hattaf, *Useful results for the qualitative analysis of generalized Hattaf mixed fractional differential equations with applications to medicine*, Computation, 13 (7), pp. 1-18 (2025).
- [2] K. Hattaf, *A new generalized definition of fractional derivative with non-singular kernel*, Computation, 8, pp. 1-9 (2020).
- [3] M. Y. Asma, M. Afzaal, M. H. DarAssi, M. A. Khan, M. Y. Alshahrani, M. Suliman, *A mathematical model of vaccinations using new fractional order derivative*, Vaccines, 10 (12), pp. 1-22 (2022).
- [4] M. R. Lemnaouar, C. Taftaf, C., Y. Louartassi, *On the controllability of fractional semilinear systems via the generalized Hattaf fractional derivative*, International Journal of Dynamics and Control, pp. 1-8 (2023).
- [5] G. Rahman, N. Mlaiki, A. Aloqaily, M. Samraiz, C. Yildiz, *Advancements in integral inequalities through Hattaf fractional operators*, Contemporary Mathematics, pp. 1110-1126 (2025).

Fractional differential equations with the general fractional derivatives with the Sonin kernels

Yuri Luchko

Department of Mathematics, Physics, and Chemistry
Berlin University of Applied Sciences and Technology, Germany

Abstract

In this talk, we first introduce general fractional integrals and derivatives with the Sonin kernels ([1]). These operators are a far-reaching generalization of the Riemann-Liouville fractional integral and derivative and build a kind of a general fractional calculus. They already found numerous applications both in mathematics and in natural sciences and in particular in probability theory, electrodynamics, continuum and statistical mechanics, linear viscoelasticity, and anomalous diffusion processes; see, e.g., [2, 3].

Then we discuss the convolution series that are generated by the Sonin kernels of the general fractional integrals and derivatives ([4]). These series are a far-reaching generalization of the power-law series. They can also be interpreted as a new representation of several important elementary and special functions including the exponential function and the famous Mittag-Leffler function that are generated by a constant and a power-law Sonin kernel, respectively.

Finally, we consider initial value problems for a class of linear fractional differential equations with general fractional derivatives with the Sonin kernels and derive their solutions explicitly in terms of the convolution series ([4, 5]). The differential equations with the Riemann-Liouville fractional derivatives are a well-known particular case of these general fractional differential equations. In this case, the solutions are expressed in terms of the Mittag-Leffler function.

References

- [1] Yu. Luchko, *General fractional integrals and derivatives and their applications* Physica D: Nonlinear Phenomena, 455, 133906 (2023).
- [2] V.E. Tarasov, *Nonlocal statistical mechanics: general fractional Liouville equations and their solutions*, Physica A: Statistical Mechanics and its Applications, 609, 128366 (2023).
- [3] K. Gorska, A. Horzela, *Subordination and memory dependent kinetics in diffusion and relaxation phenomena*, Fract. Calc. Appl. Anal., 26, pp. 480-512 (2023).
- [4] Yu. Luchko, *Special Functions of Fractional Calculus in the Form of Convolution Series and Their Applications*, Mathematics, 9(17), 2132 (2021).
- [5] Yu. Luchko, *Operational calculus for the general fractional derivative and its applications*, Fract. Calc. Appl. Anal., 24, pp. 338-375 (2021).

Modelling space inhomogeneity: Challenges and recent advances

Iwona Chlebicka

University of Warsaw, Poland

Abstract

The world we live in is not the same everywhere. Water seeps differently through sandy and clayey soils, medicines spread unevenly through various tissues in the body, and even a pot of goulash does not heat uniformly. These are everyday examples of inhomogeneous spaces – spaces whose properties vary from point to point.

Describing how diffusion or absorption takes place in such media calls for more than standard tools. One needs to understand how different regions interact and how sharp transitions between them influence the overall behavior. This gives rise to a range of deep and fascinating mathematical questions.

In this talk, I will discuss some of the core ideas that emerge in the mathematical study of inhomogeneity. A key theme will be the approximation of arbitrary functions from certain unconventional spaces by regular ones, in a way that respects the underlying energy of the system. These approximation results play a central role in the analysis of partial differential equations and variational problems that model inhomogeneous phenomena.

References

- [1] I. Chlebicka, A pocket guide to nonlinear differential equations in Musielak–Orlicz spaces, *Nonlinear Anal.* 175 (2018), 1-27.
- [2] I. Chlebicka, P. Gwiazda, A. Świerczewska-Gwiazda, A. Wróblewska-Kamińska, *Partial Differential Equations in Anisotropic Musielak–Orlicz Spaces* Springer Monographs in Mathematics, Springer Cham, 2021.
- [3] M. Borowski, I. Chlebicka, Modular density of smooth functions in inhomogeneous and fully anisotropic Musielak–Orlicz–Sobolev spaces *J. Funct. Anal.* 283 (12) (2022), 109716.
- [4] M. Borowski, I. Chlebicka, F. De Filippis, B. Miasojedow, Absence and presence of Lavrentiev’s phenomenon in double phase functionals upon any choice of exponents *Calc. Var. PDEs* (2024) 63:35.
- [5] M. Borowski, P. Bousquet, I. Chlebicka, B. Lledos, B. Miasojedow, Discarding Lavrentiev’s Gap in Non-autonomous and Non-Convex Variational Problems, preprint.

Recent Advances in Matrix Inequalities: Progress and Problems

Mohammad Sababheh

Abdullah Al Salem University, Kuwait

Princess Sumaya University for Technology, Jordan

Abstract

The numerical radius and spectral norm are two basic notions in Matrix Analysis, with a vast progress in recent years.

In this talk, we introduce the audience to some recent advances in this direction, where further accurate relations between the two notions have been proposed by numerous researchers.

While many complicated forms have been established, we will confine ourselves to simple forms that can be better understood by non-specialists.

Moreover, we present some problems in this field with our expectation of the possible outcomes. This will serve researchers who are looking for some challenging problems in the field.

The talk is designed to be accessibly to people with basic knowledge about matrices; however, the significance and importance of the discussed relations and problems need more insight into this field.

References

- [1] S. Abu Sammour, F. Kittaneh and M. Sababheh, *A geometric approach to numerical radius inequalities*, Linear Algebra Appl. 652, pp. 1-17 (2022), .
- [2] T. Ando, F. Hiai, *Operator log-convex functions and operator means*, Math. Ann. 350, pp. 611–630 (2011).
- [3] Y. Bedrani, F. Kittaneh, and M. Sababheh. *From positive to accretive matrices*, Positivity, 25(4), pp. 1601–1629 (2021).
- [4] S. Drury, *Principal powers of matrices with positive definite real part*, Linear Multilinear Algebra, 63, pp. 296–301 (2015).
- [5] S. Drury, *A numerical radius inequality for sector operators*, Linear Algebra Appl. 687, pp. 108–116 (2024).
- [6] F. Kittaneh, *A numerical radius inequality and an estimate for the numerical radius of the Frobenius companion matrix*, Studia Math., 158(1), pp. 11–17 (2003).
- [7] H. R. Moradi and M. Sababheh, *New estimates for the numerical radius*, Filomat. 35(14), pp. 4957–4962 (2021).
- [8] M. Raïssouli, M. S. Moslehian, and S. Furuichi, *Relative entropy and Tsallis entropy of two accretive operators*, C. R. Acad. Sci. Paris Ser. I., 355, pp. 687–693 (2017).

Comprehensive convergence analysis for the Adam optimizer

Arnulf Jentzen

School of Data Science and Shenzhen Research Institute of Big Data, The Chinese University of Hong Kong, Shenzhen (CUHK-Shenzhen), China

Applied Mathematics: Institute for Analysis and Numerics, Faculty of Mathematics and Computer Science, University of Münster, Germany

Abstract

Stochastic gradient descent (SGD) optimization methods are nowadays the method of choice for the training of deep neural networks (DNNs) in artificial intelligence systems. In practically relevant training problems, usually not the plain vanilla standard SGD method is the employed optimization scheme but instead suitably accelerated and adaptive SGD optimization methods such as the famous Adam optimizer are applied. In this work we establishing optimal convergence rates for the Adam optimizer covering the situation of strongly convex stochastic optimization problems (SOPs). The key ingredient of our convergence analysis is a new vector field function which we propose to refer to as the Adam vector field. This Adam vector field accurately describes the macroscopic behaviour of the Adam optimization process but differs from the negative gradient of the objective function (the function we intend to minimize) of the considered stochastic optimization problem. In particular, for a class of simple quadratic SOPs we disprove that Adam converges to critical points of the objective function (zeros of the gradient of the objective function) of the considered optimization problem. We also establish higher order convergence rates and advanced stability properties for Adam. The talk is based on joint works with Steffen Dereich, Thang Do, Robin Graeber, and Adrian Riekert.

References

- [1] S. Dereich & A. Jentzen, Convergence rates for the Adam optimizer, arXiv:2407.21078 (2024), 43 pages.
- [2] S. Dereich, R. Graeber, & A. Jentzen, Non-convergence of Adam and other adaptive stochastic gradient descent optimization methods for non-vanishing learning rates, arXiv:2407.08100 (2024), 54 pages.
- [3] S. Dereich, A. Jentzen, and A. Riekert, Sharp higher order convergence rates for the Adam optimizer, arXiv:2504.19426 (2025), 27 pages.
- [4] T. Do, A. Jentzen, & A. Riekert, Non-convergence to the optimal risk for Adam and stochastic gradient descent optimization in the training of deep neural networks, arXiv:2503.01660 (2025), 42 pages.
- [5] A. Jentzen & A. Riekert, Non-convergence to global minimizers for Adam and stochastic gradient descent optimization and constructions of local minimizers in the training of artificial neural networks, arXiv:2402.05155 (2024), 36 pages, to appear in SIAM/ASA J. Uncertain. Quantif.

Continuous data assimilation for the equation
arising as a singular limit of fast rotating 3-d
compressible viscous fluid

Piotr Gwiazda

Polish Academy of Sciences, Poland

Abstract

We consider a system of "2,5-d" equations arising as a singular limit of fast rotating 3-d compressible viscous fluid driven by temperature gradient. This singular limit was proven recently by F. Fanelli and E. Feireisl with the help of relative entropy method, which provide quantitative estimate on the distance of primitive and target system. We show the problem admits global-in-time solutions and a compact trajectory attractor. Finally, we address the problem of continuous data assimilation for the target system.

Mean oscillation conditions for nonlinear equation and regularity results

Peter Hästö

University of Helsinki, Finland

Abstract

I will present results from a recent eponymous preprint with Mikyoung Lee and Jihoon Ok. We consider general nonlinear elliptic equations of the form

$$\operatorname{div} A(x, Du) = 0 \quad \text{in } \Omega,$$

where $A : \Omega \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ satisfies a quasi-isotropic (p, q) -growth condition. We establish sharp and comprehensive mean oscillation conditions on $A(x, \xi)$ with respect to the x variable to obtain C^1 - and $W^{1,s}$ -regularity results. The results provide new conditions even in the standard p -growth case with coefficient $\operatorname{div}(a(x)|Du|^{p-2}Du) = 0$. Also included are variable exponent growth with and without perturbation as well as borderline double-phase growth and double-phase growth with coefficient, including coefficients and exponents of Sobolev–Lorentz type.

<https://sites.google.com/site/varexpspa/>
<https://www.problemsolving.fi/>